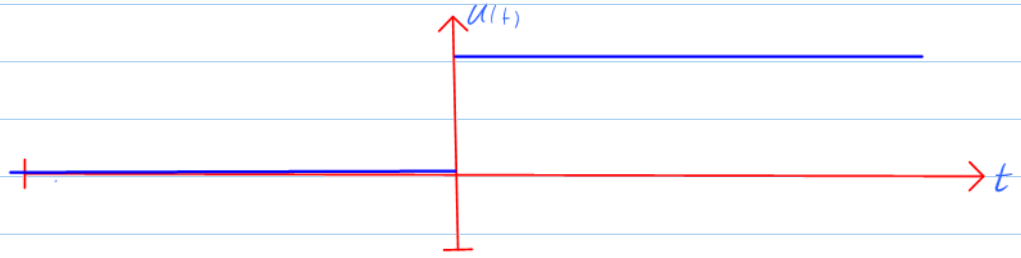
جلسه ۹ سیگنال

سیگنال‌های پایه

$$u(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases} \qquad u[n]=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$$

پله واحد ← u ①



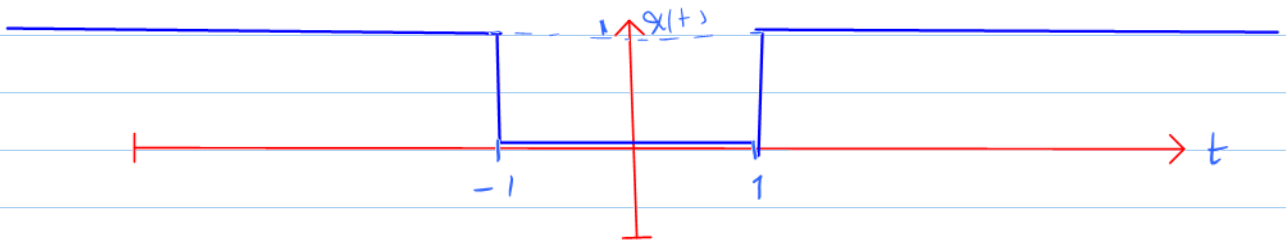
$$u(f(t))=\begin{cases}1 & f(t)\geq 0\\ 0 & f(t)<0\end{cases} \qquad u[f(n)]=\begin{cases}1 & f(n)\geq 0\\ 0 & f(n)<0\end{cases}$$

$$x(t)=u(t^2-1)=\begin{cases}1 & t^2-1\geq 0\\ 0 & t^2-1<0\end{cases}$$

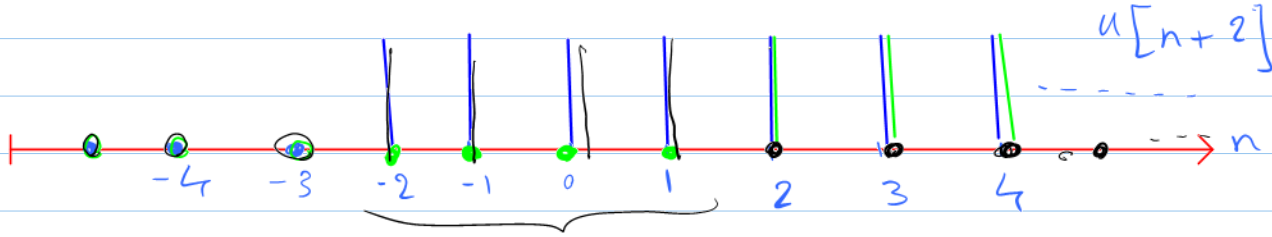
$t^2-1\geq 0 \longrightarrow t^2\geq 1 \longrightarrow |t|\geq 1 \longrightarrow \boxed{t\geq 1 \text{ یا } t\leq -1}$

$t^2-1<0 \longrightarrow t^2<1 \longrightarrow |t|<1 \longrightarrow -1<t<1$

$$x(t)=u(t^2-1)=\begin{cases}1 & t\geq 1 \text{ یا } t\leq -1\\ 0 & -1<t<1\end{cases}$$



$$x[n]=u[n+2]-u[n-2]$$



$$u[n+2] = \begin{cases} 1 & n+2 \geq 0 \rightarrow n \geq -2 \\ 0 & n+2 < 0 \rightarrow n < -2 \end{cases}$$

$$u[n-2] = \begin{cases} 1 & n-2 \geq 0 \rightarrow n \geq 2 \\ 0 & n-2 < 0 \rightarrow n < 2 \end{cases}$$

نحوه در انتگرال یا سیگما:

هرگاه انتگرال شامل u دیدید، اول u را صفر کنید.

$$\int_{2}^{\infty} f(t)\, \underbrace{u(t-2)}_{t \geq 2}\, dt = \int_{2}^{\infty} f(t)\, dt$$

$$\int_{-\infty}^{\infty} f(t)\, \underbrace{\left(u(t+2) - u(t-1)\right)}_{-2 \leq t \leq 1}\, dt$$

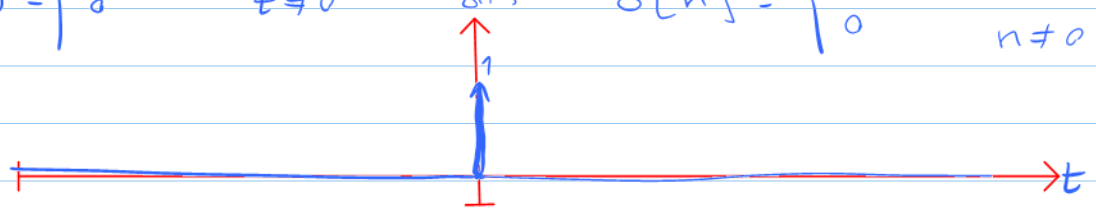
$$u(t+a) - u(t-b) = \begin{cases} 1 & -a \leq t \leq b \\ 0 & \text{else} \end{cases}$$

$$\int_{-2}^{1} f(t)\, \underbrace{\left(u(t+2) - u(t-1)\right)}_{-2 \leq t \leq 1}\, dt = \int_{-2}^{1} f(t)\, dt$$

$$\sum_{k=10}^{\infty} f(k)\, \underbrace{u(k-10)}_{k \geq 10} = \sum_{k=10}^{\infty} f(k)$$

② ضربه δ:

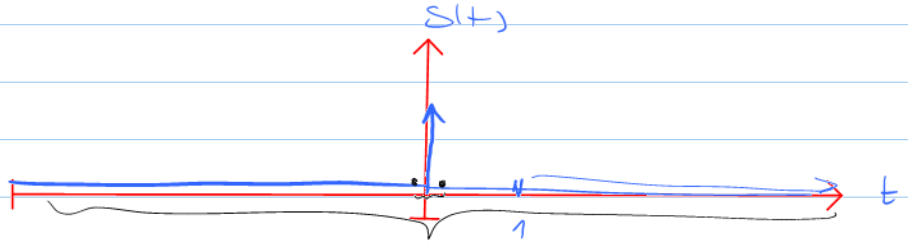
$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases} \qquad \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



$$\delta(f(t)) = \begin{cases} 1 & f(t)=0 \\ 0 & f(t)\neq 0 \end{cases}$$

$$\delta(2t-4) = \begin{cases} 1 & 2t-4=0 \rightarrow t=2 \\ 0 & 2t-4\neq 0 \rightarrow t\neq 2 \end{cases}$$

خواص ضربه:



1- انتگرال ضربه

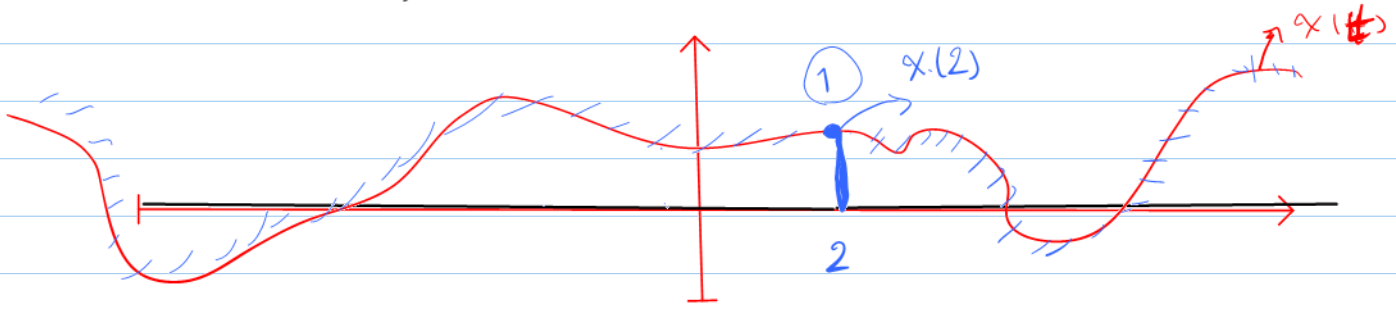
$$\int_b^c \delta(t-a)\,dt = \begin{cases} 1 & a\in[b,c] \\ 0 & a\notin[b,c] \\ & 0\notin[1,\infty] \end{cases}$$

$$\int_1^{\infty} \delta(t)\,dt = 0 \qquad \int_{-\infty}^{2} \delta(t-1)\,dt = 1$$

$$\sum_{n=-2}^{3} \delta[n-2] = 1$$

2- خاصیت غربالگری:

$$x(t)\times\delta(t-2) = x(2)\,\delta(t-2)$$



$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) \qquad (t=t_0)$$

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0] \qquad (n=n_0)$$

مثال:

$$\cos\left(\frac{\pi}{2}k\right)\delta[k-1] = \cos\left(\frac{\pi}{2}\times 1\right)\delta[k-1] = 0$$

$k=1$

هرجایی در δ ضرب شود، اول ضابطه عزیزگری اعمال می‌شود.

$$\delta(t-2)\,\delta(t-10) = \delta(8)\,\delta(t-10) = 0$$

x ، $t=10$

$$\delta(\underset{t}{8}) = \begin{cases} 1 & t=0 \\ 0 & t\neq 0 \end{cases} = 0$$

$$\delta(t-2)\,\delta(t-2) = 1\cdot\delta(t-2) = \delta(t-2)$$

$t=2$

نکته: در انتگرال یا سیگما، هرگاه δ دیدید ابتدا ضابطه عزیزگری را اعمال کنید.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-2)\,dt = \int_{-\infty}^{\infty} x(2)\,\delta(t-2)\,dt$$

$t=2$

$x(2)$ مقدار x در لحظه 2 ثانیه

$$= x(2)\int_{-\infty}^{\infty}\delta(t-2)\,dt = x(2)\times 1 = x(2)$$

(2)

تمرین ۹۲. اگر $x_1(t)=\sum_{k=-\infty}^{\infty}\cos\left(\frac{t}{3}\right)\delta(t-k\pi)$ و $x_2(t)=\sum_{k=-\infty}^{\infty}\cos(\pi t^2)\,\delta(t-k)$

و $x_3(t)=x_1(t)+x_2(t)$ باشد در این صورت:

۱) x_1 و x_2 متناوب ولی x_3 نامتناوب

2) هر سه متناوب

۳) x_1 متناوب و x_2 و x_3 نامتناوب

۴) هر سه نامتناوب

حل دانشجو

$$x_1(t) = \sum_{k=-\infty}^{\infty} \cos\left(\frac{t}{3}\right) \delta(t - k\pi)$$

$t = k\pi$

$$x_1(t) = \sum_{k=-\infty}^{\infty} \cos\left(\frac{\pi}{3}k\right) \delta(t - k\pi)$$

$\sum_{k=-\infty}^{\infty} f(k)\, \mathbb{Z}(t - kT_0)$ یادآوری: MxT_0 تناوبی با

$f(k)$ ↓

M

$f(k) = \cos\left(\frac{\pi}{3}k\right) \rightarrow N_{0\ \text{اولیه}} = \frac{2\pi}{\pi/3} = 6$ ✓

$f(k)$ ، $M = 6$

$T_0 = \pi \rightarrow x_1(t)$ ، $6 \times \pi$

x_1 تناوبی با 6π است.

$$x_2(t) = \sum_{k=-\infty}^{\infty} \cos(\pi t^2)\, \delta(t - k)$$

$t = k$

$$x_2(t) = \sum_{k=-\infty}^{\infty} \underbrace{\cos(\pi k^2)}_{f(k)}\, \delta(t - k)$$

$T_0 = 1$

اگر سیگنال گسسته $f(k)$ تناوبی با M باشد ...

$\cos(\pi k^2) \rightarrow N_0 = ?$

$\cos(\pi k) \rightarrow N_1 = \frac{2\pi}{\pi} = 2$

k^2 ↓

$\cos(\pi k^2) \rightarrow N_0 = 2$

$x_2(t) \rightarrow MxT_0 = 2 \times 1 = 2$

$x_3(t) = x_1(t) + x_2(t)$

T_0 ← 6π ← 2

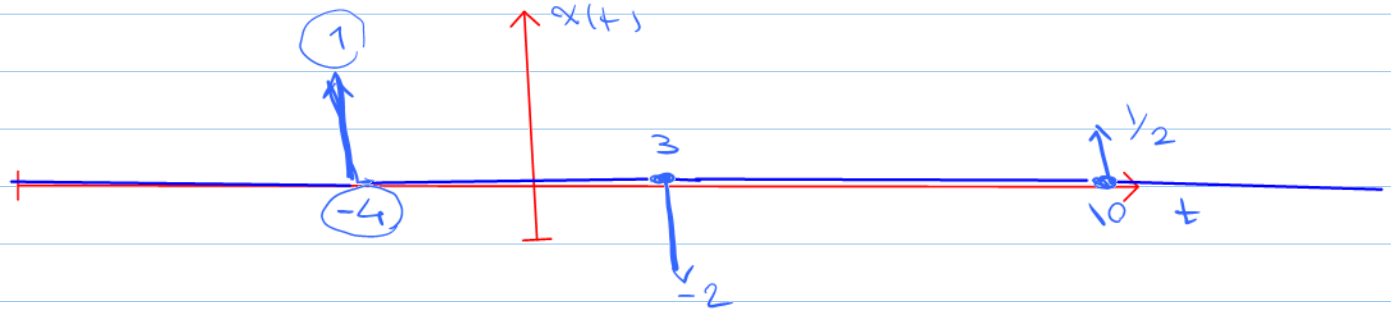
x_3 غیرتناوبی

وجود ندارد $= T_0 =$ ک.م.م $\{6\pi, 2\}$

3- جمع ضربه‌ها

$(a+b)\,\delta(t) = a\,\delta(t) + b\,\delta(t)$

جهت پیکانی را برحسب ضربه بنویسید:



$x(t) = -1\,\delta(t-(-4)) + -2\,\delta(t-3) + \frac{1}{2}\,\delta(t-10)$

4- مقیاس دهی

$\delta(at) = \frac{1}{|a|}\,\delta(t)$

(*) $\delta(a\,f(t)) = \frac{1}{|a|}\,\delta(f(t))$

$\delta(2t) = \frac{1}{|2|}\,\delta(t)$



(*) $\delta[a f[n]] = \delta[f[n]]$

حل:

$\delta[2n] = \delta[n]$

$\delta[2n-2] = \delta[n-1]$

$$\int_{-\infty}^{\infty} x(\tau)\,\delta^{(n)}(\tau - t_0)\,d\tau = ?$$ -5

انتگرالی از مشتق ضربه:

$$\int_{-\infty}^{\infty} x(\tau - t_0)\,\delta^{(n)}(\tau)\,d\tau = (-1)^n \left.\frac{d^n x(t)}{dt^n}\right|_{t=t_0}$$

$$\delta(f(t)) = \sum_i \frac{\delta(t - t_i)}{|f'(t)|_{t=t_i}|}$$ -6

$\delta(t^2 - 1)$ و $\delta(\cos(t)(t-2)) = ?$

t_iها ریشه‌های $f(t)$ هستند.

$\delta(\underbrace{8t - t^3}_{f(t)}) = ?$ برحسب ضابطه‌ی ضربه بنویسید.

① ریشه‌های $f(t)$: $8t - t^3 = 0$

$$\begin{cases} t_1 = 0 \\ t_2 = \sqrt{8} \\ t_3 = -\sqrt{8} \end{cases}$$

$$\delta(8t - t^3) = \sum_{i=1}^{3} \frac{\delta(t - t_i)}{|f'(t_i)|}$$

$$f'(t) = 8 - 3t^2$$

$$\delta(8t - t^3) = \frac{\delta(t-0)}{|8 - 3\times 0^2|} + \frac{\delta(t - \sqrt{8})}{|8 - 3(\sqrt{8})^2|} + \frac{\delta(t - (-\sqrt{8}))}{|8 - 3(-\sqrt{8})^2|}$$

$$\delta(8t - t^3) = \frac{\delta(t)}{8} + \frac{\delta(t-\sqrt{8})}{16} + \frac{\delta(t+\sqrt{8})}{16}$$

هرگاه داخل δ عبارتی به غیر $at \mp b$ باشد از خاصیت (6) استفاده می‌کنیم.

$$\delta(at \mp b)$$

مثال ۹. حاصل انتگرال زیر که در آن δ تابع ضربه و δ' مشتق آن است، کدام است؟ 2 ۰ ۱ -۱

$$I = \int_{-\infty}^{\infty} \left[(t+2)\,\delta'(t+1) + (e^{-|t|} + t^2 + 2)\,\delta(e^{-|t|} + t^2 + 1) \right] dt$$

$$I = \int_{-\infty}^{\infty} (t+2)\,\delta'(t+1)\,dt + \int_{-\infty}^{\infty} (e^{-|t|}+t^2+2)\,\delta(e^{-|t|}+t^2+1)\,dt$$

$$\int_{-\infty}^{\infty} \underbrace{(t+2)}_{x(t)}\,\delta^{(1)}(t+1)\,dt = (-1)^1\,1 = -1$$

n, $-t_0$, $t_0 = -1$

$t+1 = t-(-1)$, t_0

$$\int_{-\infty}^{\infty} x(\tau)\,\delta^{(n)}(\tau - t_0)\,d\tau = (-1)^n\,x^{(n)}(t_0)$$

$$x'(\tau) = 1$$

$$\int_{-\infty}^{\infty} (e^{-|t|}+t^2+2)\,\underbrace{\delta(e^{-|t|}+t^2+1)}\,dt$$

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

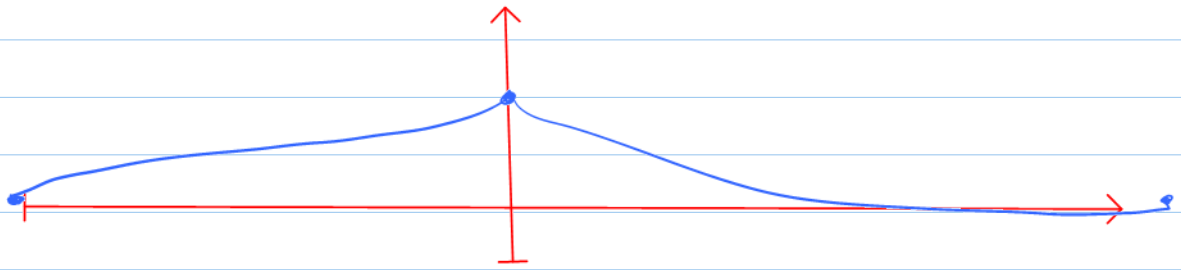
$$\delta(at \mp b) \rightarrow$$

$$\delta(f(t)) = \sum_i \frac{\delta(t-t_i)}{|f'(t_i)|}$$

$$\delta(\underbrace{e^{-|t|}+t^2+1}_{f(t)}) = ?$$

$$e^{-|t|}+t^2+1 = 0 \rightarrow t = ?$$

$e^{-|t|}$, t^2 , 1



$e^{-|t|} > 0$ هردو (مثبت) > 0

$t^2 \geq 0 \rightarrow t^2+1 \geq 1$

$1 > 0$

$$e^{-|t|} + t^2 + 1 \geq 1$$

$$\delta(e^{-|t|} + t^2 + 1) = 0$$

$$\delta(F(t)) = \begin{cases} \infty & F(t) = 0 \\ 0 & F(t) \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} (e^{-|t|} + t^2 + 2) \times 0 \, dt = 0$$

$$I = -1 + 0 = -1$$

ارتباط ضربه و پله :

1) $\frac{du(t+a)}{dt} = \delta(t+a)$

ضربه = مشتق پله

2) $u[n+a] - u[n+a-1] = \delta[n+a]$

$u[n] - u[n-1] = \delta[n]$

3) $\int_{\alpha}^{\beta} \delta(t-t_0)\,dt = u(\beta - t_0) - u(\alpha - t_0)$

$\sum_{k=m_1}^{m_2} \delta(n-n_0) = u[m_2 - n_0] - u[m_1 - 1 - n_0]$

الف) $\sum_{k=-\infty}^{n-2} (k+2)\,\delta[k+1]$

ب) $\sum_{k=-\infty}^{n} 3^k\,\delta[2-k]$

ج) $\int_{t-5}^{t} z^2\,\delta(2z-6)\,dz$

$$\sum_{k=-\infty}^{n-2} (k+2)\,\delta[\underbrace{k+1}_{k=-1}] = ?$$

$$\sum_{k=m_1}^{m_2} \delta[k-n_0] = u[m_2-n_0] - u[m_1-1-n_0]$$

$$= \sum_{k=-\infty}^{n-2} (-1+2)\,\delta(k+1) = \sum_{k=-\infty}^{\overbrace{n-2}^{m_2}} \delta(\underbrace{k+1}_{n_0=-1}) \quad (k=-\infty = m_1)$$

$$= u[n-2+1] - u[-\infty + \infty]$$

$$= u[n-1] - u[-\infty] = u[n-1]$$

$$\sum_{k=a}^{b} \delta[k-c] = \begin{cases} 1 & c \in [a,b] \\ 0 & c \notin [a,b] \end{cases}$$:يُذكّر

$$\hookrightarrow) \sum_{k=-\infty}^{n} 3^k\,\delta[\underbrace{2-k}_{k=2}] = \sum_{k=-\infty}^{n} \overbrace{3^2}\,\delta[2-k] =$$

$$= 9 \sum_{k=-\infty}^{n} \delta[2-k]$$

$$= 9 \sum_{k=-\infty}^{n} \delta[k-2] = 9\left(u[n-2] - u[-\infty + 2]\right)$$

$$= 9\,u[n-2]$$

$$\delta[a f(n)] = \delta[f(n)]$$

$$\delta[\underbrace{-1(k-2)}_{2-k}] = \delta[k-2]$$

2) $\int_{t-5}^{t} \tau^2 \delta(2\tau - 6)\, d\tau$ ($\tau = 3$)

$$\int_{\alpha}^{\beta} \delta(t - t_0)\, dt = u(\beta - t_0) - u(\alpha - t_0)$$

$$= \int_{t-5}^{t} 9\,\delta(2\tau - 6)\, d\tau = 9\int_{t-5}^{t} \delta(2(\tau - 3))\, d\tau$$

$$\delta(a f(t)) = \frac{1}{|a|}\delta(f(t))$$

$$\delta(2(\tau - 3)) = \frac{1}{|2|}\delta(\tau - 3)$$

$$= 9\int_{t-5}^{t} \frac{1}{2}\delta(\tau - 3)\, d\tau = \frac{9}{2}\int_{t-5}^{t} \delta(\tau - 3)\, d\tau$$

($\alpha = t-5$, $\beta = t$)

$$= \frac{9}{2}\left(u(t-3) - u(t-5-3)\right)$$

$$= \frac{9}{2}\left(u(t-3) - u(t-8)\right) \checkmark$$

آزاد ۳۷ . تابع $e^{-t}\delta'(t)$ برابر کدام گزینه است؟

$\delta'(t)$ $\quad$ $\delta'(t) + \delta(t)$ $\quad$ $-\delta(t)$ $\quad$ 0

$$\frac{d}{dt}\left(f(t)g(t)\right) = f'(t)g(t) + g'(t)f(t)$$

$$e^{-t}\delta'(t) = ?$$

$$e^{-t}\delta(t) = e^{-0}\delta(t)$$

$t=0$

$$\frac{d}{dt}\left(e^{-t}\delta(t)\right) = \frac{d}{dt}\delta(t)$$

$$-e^{-t}\delta(t) + e^{-t}\delta'(t) = \delta'(t)$$

$$-\delta(t) + e^{-t}\delta'(t) = \delta'(t)$$

$$e^{-t}\delta'(t) = \delta'(t) + \delta(t)$$

آزاد ۹۸ . حاصل انتگرال $I = \int_0^{\infty} (t^2+1)\left(\delta'(t-3) + \delta(t^2-1)\right) dt$ کدام است؟

8 7 -5 -4

$$I = \int_0^{\infty} \left((t^2+1)\,\delta'(t-3) + (t^2+1)\,\delta(t^2-1) \right) dt$$

$$= \underbrace{\int_0^{\infty} (t^2+1)\,\delta'(t-3)\,dt}_{I_1} + \underbrace{\int_0^{\infty} (t^2+1)\,\delta(t^2-1)\,dt}_{I_2}$$

$$I_1 = \int_0^{\infty} \underbrace{(t^2+1)}_{x(t)}\, \overset{n=1}{\overset{\downarrow}{\delta}}(t-\underbrace{3}_{t_0})\,dt = (-1)^1 \times 2(+3) = \underline{-6}$$

$$\int x(t)\,\delta^{(n)}(t-t_0)\,dt = (-1)^n\, x^{(n)}(t_0)$$

$$x(t) = t^2+1 \qquad x'(t) = 2t$$

$$I_2 = \int_0^{\infty} (t^2+1)\,\delta(t^2-1)\,dt$$

$\delta(t^2-1)$ → $t = \pm 1$?

$$\delta(at \mp b)$$

$$\delta(f(t)) = \sum_i \frac{\delta(t-t_i)}{|f'(t_i)|}$$

$$I_2 = 2\int_0^{\infty} \delta(t^2-1)\,dt = ?$$

$t_1 = 1$

$t_2 = -1$

$$\delta(t^2-1) = \sum_{i=1}^{2} \frac{\delta(t-t_i)}{|f'(t_i)|}$$

$f(t) = t^2-1 \longrightarrow$ $t_1 = 1$, $t_2 = -1$ $\qquad f'(t) = 2t$

$$\delta(t^2-1) = \frac{\delta(t-1)}{|2\times 1|} + \frac{\delta(t-(-1))}{|2\times -1|}$$

$$\delta(t^2-1) = \frac{1}{2}\delta(t-1) + \frac{1}{2}\delta(t+1)$$

$$I_2 = \int_0^\infty (t^2+1)\left(\frac{1}{2}\delta(t-1) + \frac{1}{2}\delta(t+1)\right) dt$$

$$= \int_0^\infty \frac{1}{2}(t^2+1)\,\delta(t-1)\,dt + \int_0^\infty \frac{1}{2}(t^2+1)\,\delta(t+1)\,dt$$

($\delta(t-1)$: $t=1$; $\delta(t+1)$: $t=-1$)

$$= \int_0^\infty \delta(t-1)\,dt + \int_0^\infty \delta(t+1)\,dt$$

$$= 1 + 0 = 1$$

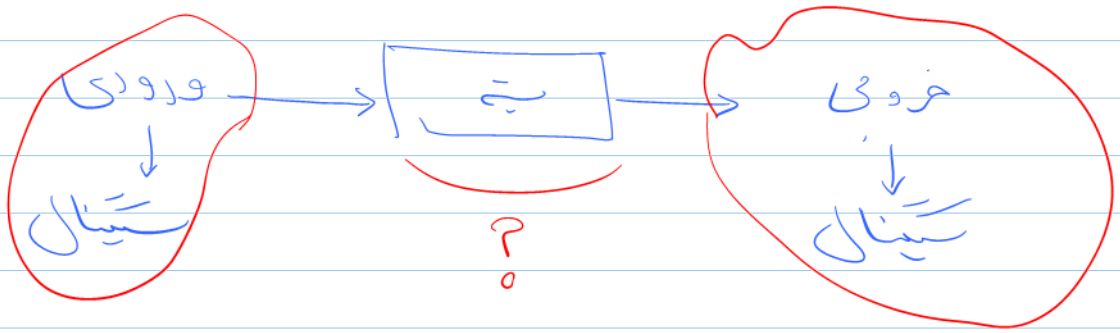
$$I = -6 + 1 = -5$$

$$2 \neq 5$$

خواص سیگنال‌ها

↓

تجزیه و تحلیل سیستم



سیستم زمان پیوسته: اگر ورودی و خروجی سیستم پیوسته باشد

سیستم گسسته: ... گسسته

خواص سیستم‌ها ← خطا

TI یا TV

حافظه

معکوس پذیری

علی

پایداری